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Relation between the $^{16}O(\alpha,\gamma)^{20}Ne$ reaction and its reverse $^{20}Ne(\gamma,\alpha)^{16}O$ reaction in stars and in the laboratory

P. Mohr[1,a], C. Angulo[2], P. Descouvemont[3], and H. Utsunomiya[4,b]

[1] Diakoniekrankenhaus, D-74523 Schwäbisch Hall, Germany
[2] Centre de Recherches du Cyclotron, UCL, Louvain-la-Neuve, Belgium
[3] PNTPM, Université Libre de Bruxelles, Brussels, Belgium
[4] Konan University, Kobe, Japan

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Abstract. The astrophysical reaction rates of the $^{16}O(\alpha,\gamma)^{20}Ne$ capture reaction and its inverse $^{20}Ne(\gamma,\alpha)^{16}O$ photodisintegration reaction are given by the sum of several narrow resonances and a small direct capture contribution at low temperatures. Although the thermal population of low-lying excited states in ^{16}O and ^{20}Ne is extremely small, the first excited state in ^{20}Ne plays a non-negligible role for the photodisintegration rate. Consequences for experiments with so-called quasi-thermal photon energy distributions are discussed.

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1 Introduction

The small reaction rate of the $^{16}O(\alpha,\gamma)^{20}Ne$ capture reaction blocks the reaction chain $3\alpha \rightarrow {}^{12}C(\alpha,\gamma)^{16}O(\alpha,\gamma)^{20}Ne$ in helium burning at typical temperatures around $T_9 = 0.2$ (T_9 is the temperature in billion degrees K). Its inverse $^{20}Ne(\gamma,\alpha)^{16}O$ photodisintegration reaction is one of the key reactions in neon burning at higher temperatures around $T_9 = 1$–2 [1]. Only at very low temperatures below $T_9 = 0.2$ the $^{16}O(\alpha,\gamma)^{20}Ne$ reaction rate is dominated by direct capture. At higher temperatures the reaction rate is given by the sum of several resonances [2]. The contributions of several low-lying resonances to the reaction rate of the $^{16}O(\alpha,\gamma)^{20}Ne$ reaction are shown in fig. 1. The properties of three bound states in ^{20}Ne and four selected low-lying resonances are listed in table 1.

Usually, the reaction rates of inverse photodisintegration reactions are calculated from the capture rates using the detailed-balance theorem which is only valid if all nuclei involved are fully thermalized (see, *e.g.*, [2]). In the case of light nuclei where the first excited states are located at relatively high energies one finds very small occupation probabilities for these excited states. The scope of this paper is to analyze the relation between the $^{16}O(\alpha,\gamma)^{20}Ne$ and $^{20}Ne(\gamma,\alpha)^{16}O$ reaction rates

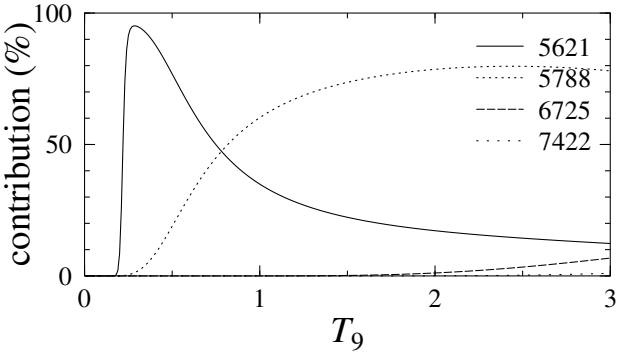

Fig. 1. Contribution of individual resonances to $^{16}O(\alpha,\gamma)^{20}Ne$ reaction rate. The resonances are labelled by their energies E_R in keV (see also table 1). At very low temperatures below $T_9 = 0.2$ direct capture is dominating.

Table 1. Properties of levels in ^{20}Ne below and above the ^{16}O-α threshold at $Q = 4730$ keV [3] (from [2,4]).

E_x (keV)	J^π	E_R^α (keV)	$(\omega\gamma)$ [4] (meV)	$(\omega\gamma)$ [2] (meV)	B^0 (%)	B^{1634} (%)
0	0^+	–	–	–	–	–
1634	2^+	–	–	–	100	–
4248	4^+	–	–	–	0	100
5621	3^-	892	1.7(3)	1.9(3)	7.6(10)	87.6(10)
5788	1^-	1058	17(3)	23(3)	18(5)	82(5)
6725	0^+	1995	71(12)	74(9)	0	100
7422	2^+	2692	146(19)	160(20)	$\leq 9.4(14)$	$\geq 90.6(14)$

[a] e-mail: WidmaierMohr@compuserve.de
[b] e-mail: hiro@konan-u.ac.jp

for the particular case of high-lying first excited states (^{16}O: $E_x = 6049\,\mathrm{keV}$, 0^+; ^{20}Ne: $E_x = 1634\,\mathrm{keV}$, 2^+). Furthermore, we discuss potential experiments using so-called quasi-thermal photon spectra [5,6] where the target nucleus is always in its ground state.

2 Resonant reaction rates

Because of the dominance of resonances in the rates of the $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ and $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ reactions we restrict ourselves to the discussion of resonances. The reaction rate $\langle\sigma\, v\rangle$ of a Breit-Wigner resonance with

$$\sigma_{\mathrm{BW}}(E) = \frac{\pi}{k_\alpha^2}\,\omega\,\frac{\Gamma_\alpha\Gamma_\gamma}{(E-E_R^\alpha)^2+\Gamma^2/4} \tag{1}$$

is given by

$$\langle\sigma\, v\rangle = \hbar^2\left(\frac{2\pi}{\mu kT}\right)^{3/2}(\omega\gamma)\,\exp\left(\frac{-E_R^\alpha}{kT}\right) \tag{2}$$

with the reduced mass μ, the resonance energy E_R^α in the c.m. system, and the resonance strength

$$(\omega\gamma) = \omega\,\frac{\Gamma_\alpha\Gamma_\gamma}{\Gamma} \tag{3}$$

and the statistical factor

$$\omega = \frac{(2J_R+1)}{(2J_1+1)(2J_2+1)}\,. \tag{4}$$

Γ_α is the α decay width of the resonance,

$$\Gamma_\gamma = \sum_b \Gamma_\gamma^b \tag{5}$$

the total radiation width (summed over all partial radiation widths Γ_γ^b to bound states b), $\Gamma = \Gamma_\alpha + \Gamma_\gamma$ the total decay width, and

$$B^b = \Gamma_\gamma^b/\Gamma_\gamma \tag{6}$$

is the γ-ray branching ratio to a bound state b.

The total astrophysical reaction rate is obtained by adding the contributions of all resonances and of a small direct contribution. In the case of the $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ reaction, all contributions in the laboratory (with the target in the ground state) and under stellar conditions are identical up to high temperatures because of the high-lying first excited state in ^{16}O [2].

The cross section of the $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ photodisintegration reaction is directly related to the capture cross section by time-reversal symmetry:

$$\frac{\sigma_{3\gamma}(E_\gamma)}{\sigma_{12}(E_\alpha)} = \frac{\lambda_\gamma^2}{\lambda_{12}^2}\,\frac{(2J_1+1)(2J_2+1)}{2\,(2J_3+1)}\,. \tag{7}$$

The cross section of a Breit-Wigner resonance for the capture reaction to a bound state b is given by

$$\sigma_{12}(E_\alpha) = B^b \times \sigma_{\mathrm{BW}}(E) \tag{8}$$

with the branching ratio B^b as defined in eq. (6). Using eq. (7), the thermal photon density from the Planck law

$$n_\gamma\,(E_\gamma,T)\,\mathrm{d}E_\gamma = \frac{1}{\pi^2}\,\frac{1}{(\hbar c)^3}\,\frac{E_\gamma^2}{\exp\,(E_\gamma/kT)-1}\,\mathrm{d}E_\gamma \tag{9}$$

and the definition of the reaction rate

$$\lambda = c\int n_\gamma(E_\gamma,T)\,\sigma(E_\gamma)\,\mathrm{d}E_\gamma\,, \tag{10}$$

one obtains a reaction rate for the target in a defined bound state b with spin J_3:

$$\lambda^b = \frac{1}{\hbar}\,\frac{(2J_1+1)(2J_2+1)}{(2J_3+1)}\,B^b\,(\omega\gamma)\,\exp\left(\frac{-E_R^\gamma}{kT}\right) \tag{11}$$

with the required photon energy

$$E_R^\gamma = E_R^\alpha + Q - E_x^b \tag{12}$$

and E_x^b is the excitation energy of the bound state b. Note that the required photon energy E_R^γ for the transition to a resonance at excitation energy E_R is reduced by the excitation energy E_x^b of the bound state b under consideration!

For simplicity we choose the example of the 1^- resonance at $E_R = 5788\,\mathrm{keV}$ in ^{20}Ne in the following discussion. This resonance has a strength of $\omega\gamma = 23\,\mathrm{meV}$ and branching ratios of $B^0 = 18\%$ to the ground state and $B^{1634} = 82\%$ to the first excited state in ^{20}Ne at $E_x = 1634\,\mathrm{keV}$ with $J^\pi = 2^+$ [2,4] (see table 1).

For the ground-state rate λ^0 we obtain

$$\lambda^0 = \frac{1}{\hbar}\,B^0\,(\omega\gamma)\,\exp\left(\frac{-E_R}{kT}\right), \tag{13}$$

whereas the rate λ^{1634} for ^{20}Ne in its first excited state is

$$\lambda^{1634} = \frac{1}{\hbar}\,\frac{1}{5}\,B^{1634}\,(\omega\gamma)\,\exp\left(\frac{-(E_R-E_x^{1634})}{kT}\right). \tag{14}$$

Note the factor of 1/5 because of $J^\pi = 2^+$ for the first excited state at $E_x = 1634\,\mathrm{keV}$. Using the thermal occupation probability ratio

$$n^{1634}/n^0 = \frac{(2J^{1634}+1)}{(2J^0+1)}\,\exp\left(\frac{-E_x^{1634}}{kT}\right) \tag{15}$$

we can calculate the total astrophysical reaction rate $\lambda^\star$ for this single narrow 1^- resonance:

$$\begin{aligned}\lambda^\star &\approx \frac{1}{\hbar}\,(\omega\gamma)\left[B^0\,\exp\left(\frac{-E_R}{kT}\right)\right.\\ &\quad\left.+\,\frac{1}{5}B^{1634}\exp\left(\frac{-(E_R-E_x^{1634})}{kT}\right)5\,\exp\left(\frac{-E_x^{1634}}{kT}\right)\right]\\ &= \frac{1}{\hbar}\,(\omega\gamma)\,\left(B^0+B^{1634}\right)\,\exp\left(\frac{-E_R}{kT}\right)\\ &= \frac{1}{\hbar}\,(\omega\gamma)\,\exp\left(\frac{-E_R^\alpha}{kT}\right)\,\exp\left(\frac{-Q}{kT}\right).\end{aligned} \tag{16}$$

Comparing eqs. (2) and (16), one finds the relation between the capture rate $\langle\sigma v\rangle$ and the photodisintegration rate $\lambda^\star$:

$$\frac{\lambda^\star}{\langle\sigma v\rangle} = \left(\frac{\mu\,kT}{2\pi\hbar^2}\right)^{3/2}\,\exp\left(\frac{-Q}{kT}\right). \tag{17}$$

This is identical to the result of the detailed-balance theorem for $J^\pi(\alpha) = J^\pi(^{16}\mathrm{O}) = J^\pi(^{20}\mathrm{Ne}) = 0^+$ and $G(\alpha) = G(^{16}\mathrm{O}) = G(^{20}\mathrm{Ne}) = 1$, where G are the temperature-dependent normalized partition functions as, *e.g.*, defined in [2]. Following [2], $G(T)$ do not deviate more than 1% from unity for α, ^{16}O, and ^{20}Ne up to $T_9 = 3$.

3 Discussion

There are several interesting aspects which arise from the calculations in sect. 2. As already pointed out, the first excited states of ^{16}O and ^{20}Ne have relatively high excitation energies. *E.g.*, at $T_9 = 1$ this leads to occupation probabilities of 3×10^{-31} for the 0^+ state in ^{16}O at $E_x = 6049\,\mathrm{keV}$ and 3×10^{-8} for the 2^+ state in ^{20}Ne at $E_x = 1634\,\mathrm{keV}$. The normalized partition functions practically do not deviate from unity up to $T_9 = 3$ [2].

The reaction rate $\langle\sigma v\rangle$ for the ^{16}O$(\alpha,\gamma)^{20}$Ne capture reaction can be determined from laboratory experiments because the rates under stellar and laboratory conditions are identical for this reaction:

$$\langle\sigma v\rangle^\star = \langle\sigma v\rangle^{\mathrm{lab}}\,. \tag{18}$$

However, this is not the case for the reaction rate of the ^{20}Ne$(\gamma,\alpha)^{16}$O photodisintegration reaction under stellar conditions and in the laboratory:

$$\lambda^\star \neq \lambda^{\mathrm{lab}}. \tag{19}$$

The stellar reaction rate $\lambda^\star$ for a resonance is given by eq. (16); the full resonance strength $(\omega\gamma)$ of the capture reaction contributes to the stellar photodisintegration rate $\lambda^\star$. The laboratory reaction rate λ^0 is reduced by the branching ratio B^0 to the ground state as can be seen from eq. (13). It is interesting to note that for the chosen example of the 1^- resonance at $E_R = 5788\,\mathrm{keV}$ the laboratory rate λ^0 is only 18% of the stellar rate $\lambda^\star$ because of the branching ratio B^0. The first excited state at $E_x = 1634\,\mathrm{keV}$ in ^{20}Ne contributes with 82% to the stellar rate $\lambda^\star$ although the thermal occupation probability is only 3×10^{-8} at $T_9 = 1$! The reason for this surprising contribution can be understood from eqs. (9) and (14). The small thermal occupation probability at $E_x = 1634\,\mathrm{keV}$ is exactly compensated by the much higher photon density at the required energy $E_R^\gamma = E_R^\alpha + Q - E_x^{1634}$.

The contribution of the laboratory reaction rate λ^{lab} to the stellar rate $\lambda^\star$ (taken from the detailed-balance calculation of [2]) is shown in fig. 2. The contributions of these resonances to the stellar ^{20}Ne$(\gamma,\alpha)^{16}$O reaction rate $\lambda^\star$ are identical to the contributions in the ^{16}O$(\alpha,\gamma)^{20}$Ne reaction rate which is shown in fig. 1.

The ratio $\lambda^\star/\langle\sigma v\rangle$ does not depend on individual properties of the respective resonance. Consequently, eq. (17) is also valid for the direct capture contribution which becomes relevant at relatively low temperatures. The detailed-balance theorem hence is also applicable to reactions between nuclei with high-lying first excited states as, *e.g.*, the ^{16}O$(\alpha,\gamma)^{20}$Ne and ^{20}Ne$(\gamma,\alpha)^{16}$O reactions. Further details on the direct capture contribution at low energies are given in [7].

A further consequence of the relation between capture and photodisintegration reactions is that the so-called Gamow windows of both reactions have a strict relation. The Gamow window —the energy region where a reaction mainly operates— of a capture reaction is characterized by its position at

$$E_{\mathrm{G}} = 1.22\left(Z_1^2 Z_2^2 A_{\mathrm{red}} T_6^2\right)^{1/3}\ \mathrm{keV} \tag{20}$$

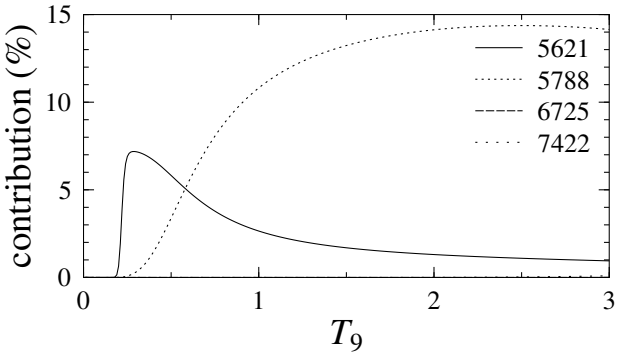

Fig. 2. Ground-state contribution $\lambda^{\mathrm{lab}}/\lambda^\star$ of individual resonances to the stellar reaction rate $\lambda^\star$ of the ^{20}Ne$(\gamma,\alpha)^{16}$O photodisintegration reaction as given by detailed balance [2]. The resonances are labelled by their energies E_R in keV. There is no contribution from the resonance at $E_R = 6725\,\mathrm{keV}$ because of $B^0 = 0$ for this resonance. Note the different scale (compared to fig. 1).

and its width Δ

$$\Delta = 0.749\left(Z_1^2 Z_2^2 A_{\mathrm{red}} T_6^5\right)^{1/6}\ \mathrm{keV}. \tag{21}$$

The corresponding Gamow window of the (γ,α) reaction is shifted by the Q value of the reaction

$$E_{\mathrm{G}}^\gamma = E_{\mathrm{G}}^\alpha + Q\,. \tag{22}$$

The width Δ remains the same for the (α,γ) and (γ,α) reactions.

In the chosen example of the ^{16}O$(\alpha,\gamma)^{20}$Ne capture and ^{20}Ne$(\gamma,\alpha)^{16}$O photodisintegration reactions, one finds, *e.g.* at $T_9 = 1$, energies $E_{\mathrm{G}}^\alpha = 1141\,\mathrm{keV}$ and $E_{\mathrm{G}}^\gamma = 5871\,\mathrm{keV}$ and a width $\Delta = 725\,\mathrm{keV}$. As an obvious consequence the 1^- resonance at $E_R = 5788\,\mathrm{keV}$ is dominating around $T_9 = 1$ (see also fig. 1). Properties of the Gamow window for (γ,α) reactions have been discussed in further detail in [8,9].

As shown above, the detailed-balance theorem holds for the ^{16}O$(\alpha,\gamma)^{20}$Ne and ^{20}Ne$(\gamma,\alpha)^{16}$O reactions provided that the population of excited states is thermal according to the Boltzmann statistics. How long does it take until thermal equilibrium is obtained in such cases where the first excited states are only weakly populated? In general, one can read from the formalism of photoactivation with a constant production rate (which is approximately fulfilled in the above case), that the time until equilibrium is reached is a few times the lifetime of the unstable product. In the case of the 2^+ state in ^{20}Ne at $E_x = 1634\,\mathrm{keV}$ the mean lifetime is $\tau = 1.05\,\mathrm{ps}$ [4]. Compared to the timescale of neon burning, which is of the order of years, thermal equilibrium is reached almost instantaneously.

4 Experiments with quasi-thermal photon spectra

Using a non-monochromatic photon spectrum, the experimental yield Y from a single resonance in the photo-

disintegration reaction $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ is given by

$$Y = N_T n_\gamma(E_R) \left(\frac{\hbar c}{E_R}\right)^2 \pi^2 B^0 \frac{(2J_1+1)(2J_2+1)}{(2J_3+1)}(\omega\gamma). \tag{23}$$

Here N_T is the number of target nuclei, and $n_\gamma(E_R)$ is the number of incoming photons per energy interval and area at the energy of the resonance E_R. Using realistic numbers for present-day facilities and thin targets for direct detection of the α-particles, one obtains a relatively small yield. *E.g.*, $N_T \approx 10^{17}$, $n_\gamma \approx 10^5/\mathrm{keV\,cm^2\,s}$, and the weak 1^- resonance at $E_R = 5788\,\mathrm{keV}$ with the properties given in table 1, leads to $Y \approx 0.5/\mathrm{d}$. For stronger resonances the yields are higher by up to two orders of magnitude which makes experiments difficult but feasible with present-day facilities.

The astrophysical reaction rate $\lambda^\star$ for the $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ photodisintegration reaction is well defined from experimental data of the $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ capture reaction. Nevertheless, experimental yields for several resonances can be obtained simultaneously in one irradiation using the quasi-thermal spectrum which will become available at SPring-8 [6]. Consequently the ratios of all observed resonances will be measured in one irradiation —provided that the ground-state branching ratios are well known. This might help to resolve minor discrepancies between the adopted resonance strengths in [4] and [2].

There are further interesting experimental properties of the $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ reaction which have to be discussed in the following. The analysis of experiments with non-monochromatic photons (see, *e.g.*, refs. [5,10,11,12]) requires the precise knowledge of the absolute number of incoming photons and their energy dependence. Using a non-monochromatic photon spectrum in combination with the $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ reaction (or any other photodisintegration reaction which is dominated by narrow resonances) one finds emitted α-particles with discrete energies. Because the extremely high-lying first excited state in $^{16}\mathrm{O}$ is practically not populated in the $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ reaction, the α energy is given by the difference between the resonance energy E_R and the Q value of the reaction. The experimental yield in each of the discrete lines is directly proportional to the resonance strength $\omega\gamma$, the ground-state branching B^0, and the number of incoming photons $n_\gamma(E_R)$ at resonance energy, as can be read from eq. (23). Provided that the resonance strengths and the branching ratios are well known, a $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ experiment may help to determine the properties of the incoming-photon spectrum with good accuracy over a broad energy range. This is especially relevant for the measurement of (γ,α) reaction rates because the relevant energy region, the so-called Gamow window, is much broader than in the case of (γ,n) reactions which have mainly been analyzed in the last years. The intrinsic exponential decrease of photon intensity with energy for the photon source suggested in [6] may help to avoid the problems of reproducing the thermal photon distribution over a broad energy range which arise in the present technique using a superposition of bremsstrahlung spectra [5].

5 Conclusions

The relation between the $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ capture reaction and the $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ photodisintegration reaction has been discussed in detail. Whereas the stellar reaction rates of the $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ reaction are identical to the laboratory rates [2], this is not the case for the photodisintegration rates under stellar and under laboratory conditions. Although the thermal population of the first excited state in $^{20}\mathrm{Ne}$ remains extremely small at typical temperatures of neon burning, it nevertheless provides an important contribution to the reaction rate under stellar conditions. The reason for this surprising behavior is that the increasing number of thermal photons at the relevant energy exactly compensates the small thermal occupation probability according to the Boltzmann statistics. The widely used detailed-balance theorem, which relates reaction rates of capture reactions to photodisintegration rates, remains valid for the case of the $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ and $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ reactions.

Additionally, it has been shown that the $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ reaction may be helpful in calibrating new intense non-monochromatic photon sources as, *e.g.*, suggested in [6]. Such a calibration over a broad energy interval is especially relevant for (γ,α) experiments because of the broader Gamow window.

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